Signals and Systems
E-623

Lecture 1
Introduction

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Course Info

Title	Signals and Systems
Code	E-623
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Course Webpage	http://www.bu.edu.eg/staff/basem.mamdoh-courses/13712
References	Multiple references will be used
Software Packages	Matlab (M-files & Simulink)

Course Aims

- To introduce the mathematical tools for analysing signals and systems in the time and frequency domain and to provide a basis for applying these techniques in engineering

- Mathematical understanding and **Matlab/Simulink-based** application
- Analyse both continuous time and discrete time signals and systems
- Analysis performed in both time and frequency domain
- Tools can be used for communications and control

Course Contents

1. Signals and Systems: Definitions – Classification – Properties
2. Fourier Series: Definition - Properties – Applications
3. Introduction to Fourier Transform.
4. Introduction to Laplace-Transforms.
5. Time-domain modeling and analysis of LTI systems.
6. Filters
7. Introduction to Z-Transforms.

Introducing Matlab

1. M-files Programing
2. Simulink

Signals Models

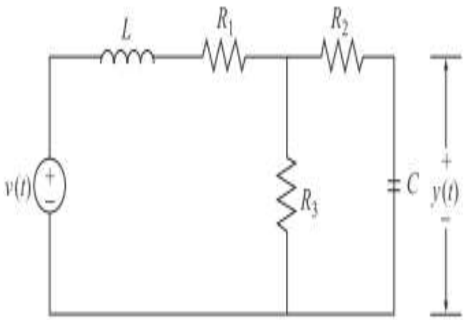
- **Signal is a function of time that represent the evolution of variable**
- Signal is a pattern of variation of some form
- Signal is variable that carry information

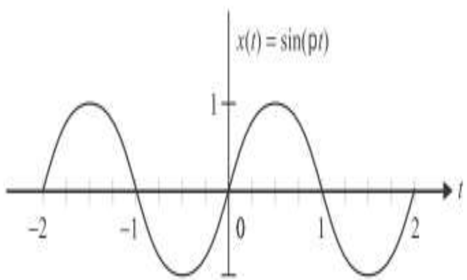
Examples of signal include:

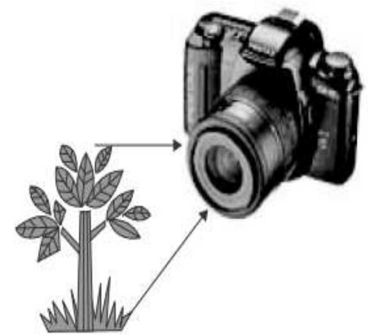
Electrical signals
  –Voltages and currents in a circuit
Acoustic signals
  –Acoustic pressure (sound) over time
Mechanical signals
  –Velocity of a car over time
Video signals
  –Intensity level of a pixel (camera, video) over time

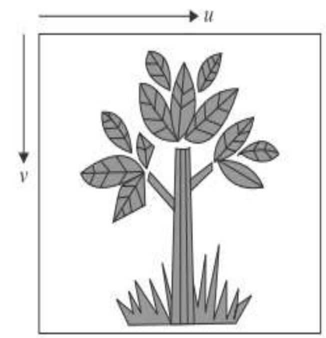
How is a Signal Represented?

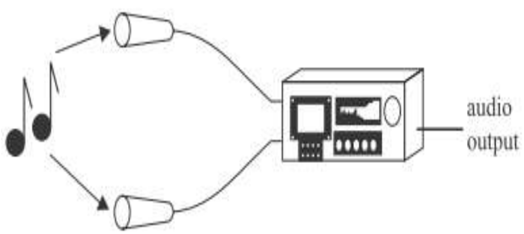
- Mathematically, signals are represented as a function of one or more **independent variables**.











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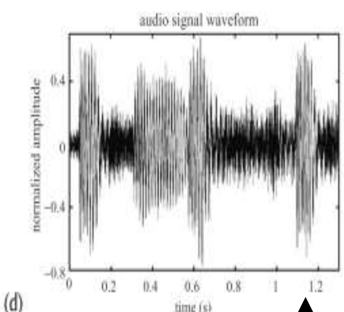

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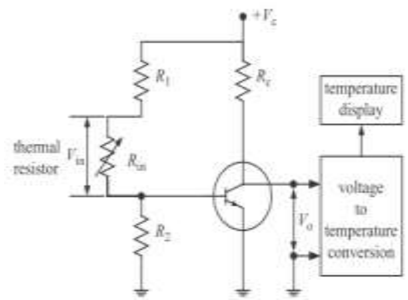
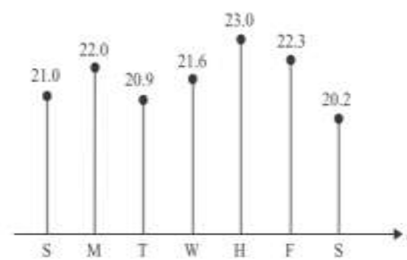
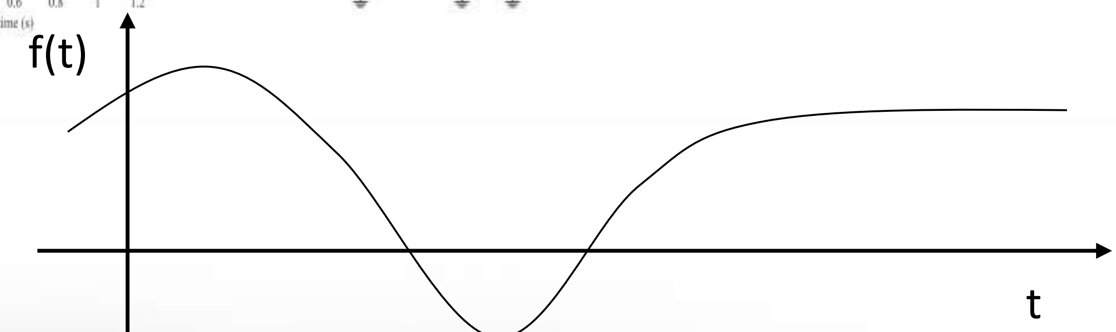




On this course, we shall be exclusively concerned with signals that are a function of a single variable: time



How is a Signal Represented?

Signals Described in Math Form:

- Consider the network of Figure 1.1 where the switch is closed at time t = 0
- We wish to describe V_{out} in a math form for the interval:

$$-\infty < t < +\infty$$

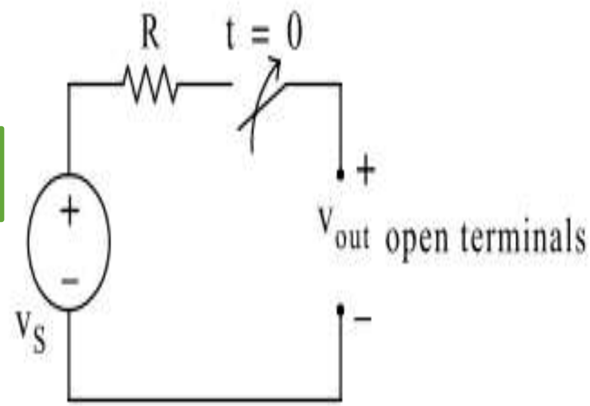


Figure 1.1. A switched network with open terminals

- We should divide the time interval into 2 parts:

For the time interval $-\infty < t < 0$, the switch is open and therefore,

$$v_{out} = 0 \quad \text{for} \quad -\infty < t < 0 \tag{1.1}$$

For the time interval $0 < t < \infty$, the switch is closed.

Then, the input voltage v_S appears at the output, i.e.,

$$v_{out} = v_S \quad \text{for} \quad 0 < t < \infty \tag{1.2}$$

How is a Signal Represented?

➢ Signals Described in Math Form:

- By combining both equations, we could get:

$$v_{out} = \begin{cases} 0 & -\infty < t < 0 \\ v_S & 0 < t < \infty \end{cases} \qquad (1.3)$$

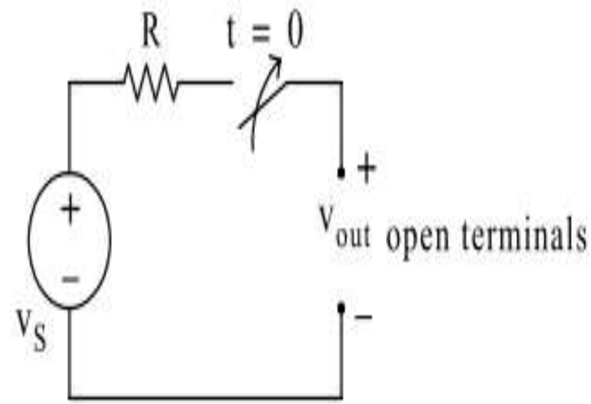


Figure 1.1. A switched network with open terminals

We can express (1.3) by the waveform shown in Figure 1.2.

- ➢ This waveform is an example of a **discontinuous** function.
- ➢ A function is said to be discontinuous if it exhibits points of discontinuity, that is, the function jumps from one value to another without taking on any intermediate values.

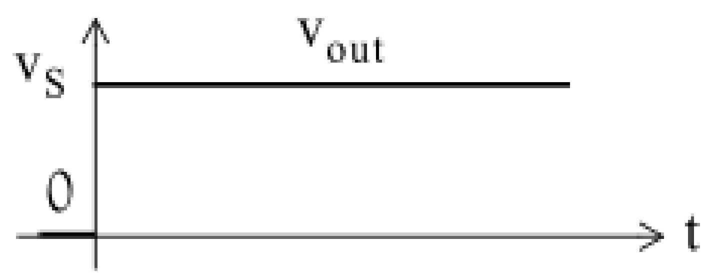


Figure 1.2.

How is a Signal Represented?

- This signal is very similar to a well-known discontinuous function in communication engineering.

The Unit Step Function $u_0(t)$

$$u_0(t) = \begin{cases} 0 & t < 0 \\ 1 & t > 0 \end{cases}$$

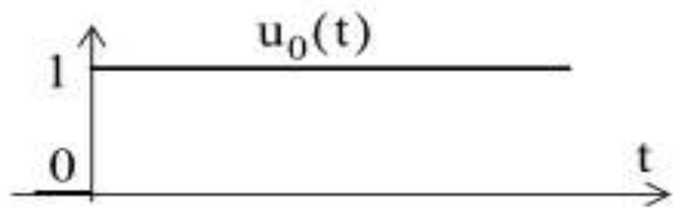


Figure 1.3. Waveform for $u_0(t)$

A waveform that changes abruptly from 0 to 1 at t = 0

- **What is the difference if the waveform changes at t = t_0 instead of t =0**

$$u_0(t - t_0) = \begin{cases} 0 & t < t_0 \\ 1 & t > t_0 \end{cases}$$

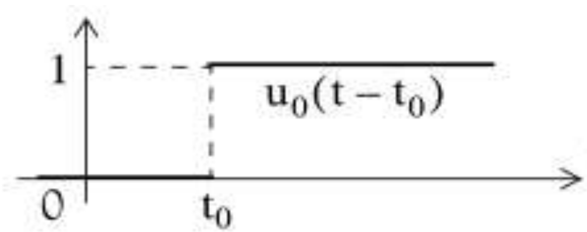


Figure 1.4. Waveform for $u_0(t - t_0)$

How is a Signal Represented?

The Unit Step Function $u_0(t)$

If the unit step function changes abruptly from 0 to 1 at $t = -t_0$

$$u_0(t+t_0) = \begin{cases} 0 & t < -t_0 \\ 1 & t > -t_0 \end{cases}$$

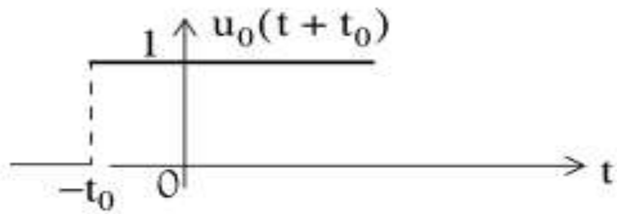


Figure 1.5. Waveform for $u_0(t+t_0)$

- **This is the "Time-Shift" property of signals**

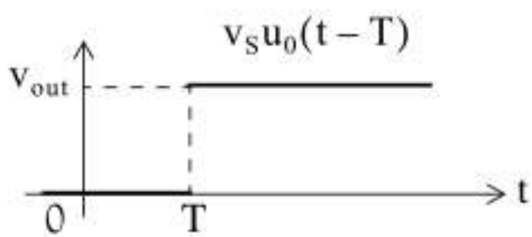


Figure 1.7. Waveform for Example 1.1

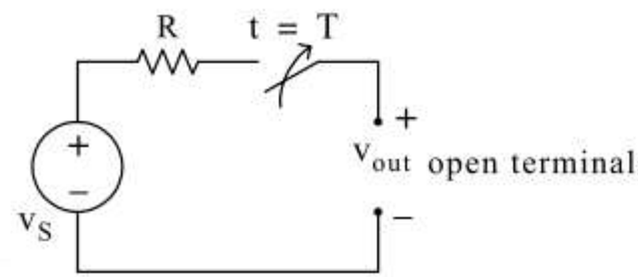


Figure 1.6. Network for Example 1.1

CLASSIFICATION OF SIGNALS

- A signal is classified into several categories depending upon the criteria used for its classification.

1. Deterministic or Random

Deterministic Signals

- If the value of a signal can be predicted for all time in advance without any error, it is referred to as a deterministic signal.
- Deterministic signals can generally be expressed in a mathematical, or graphical, or tabular form.

t	*x(t)*
0	0
1	5
2	8
3	10
4	8
5	5

$$x_1(t) = 5\sin(20\pi t + 6);$$

$$x_3(t) = \begin{cases} e^{j4\pi t} & |t| < 5 \\ 0 & \text{elsewhere;} \end{cases}$$

CLASSIFICATION OF SIGNALS

1. Deterministic or Random

Random Signals

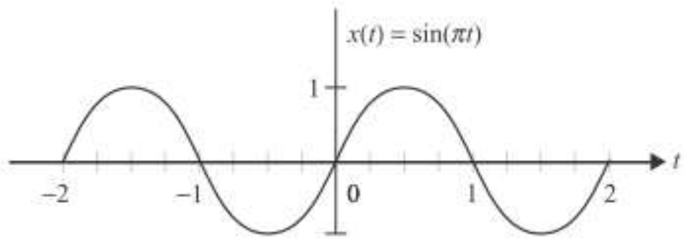
- ✓ Conversely, signals whose values cannot be predicted with complete accuracy for all time are known as **random signals**.
- ✓ Random signals cannot be modeled precisely and generally characterized by statistical measures such as means, standard deviations, and mean squared values.

Examples

- ✓ received signal due to random noise –
- ✓ thermal noise generated by a resistor.

2. Continuous-Time (CT) and Discrete-Time (DT) Signals

- ✓ If a signal is defined for all values of the independent variable t, it is called a continuous-time (CT) signal.



(a)

- CT signal is denoted by x(t) with regular parenthesis

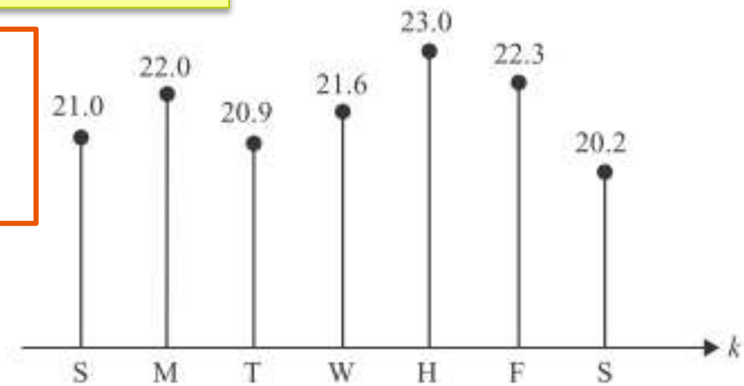
CLASSIFICATION OF SIGNALS

2. Continuous-Time (CT) and Discrete-Time (DT) Signals

- ✓ if a signal is defined only at discrete values of time, it is called a discrete-time (DT) signal.
- ✓ DT is denoted with square parenthesis as follow:

$$x[kT], \quad k = 0, \pm 1, \pm 2, \pm 3, \ldots,$$

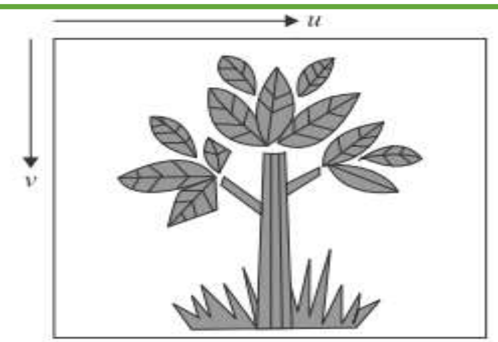
where T denotes the time interval between two consecutive samples.



- temperature of a room measured at the same hour every day for one week.
- No information is available for the temperature in between the daily readings.

- we denote a one-dimensional (1D) DT signal x by x[k]

- we denote a Two-dimensional (2D) DT signal x by x[m,n] like the output of CCD camera



2. Continuous-Time (CT) and Discrete-Time (DT) Signals

Example 1.1

Consider the CT signal $x(t) = \sin(\pi t)$ plotted in Fig. 1.3(a) as a function of time t. Discretize the signal using a sampling interval of $T = 0.25$ s, and sketch the waveform of the resulting DT sequence for the range $-8 \le k \le 8$.

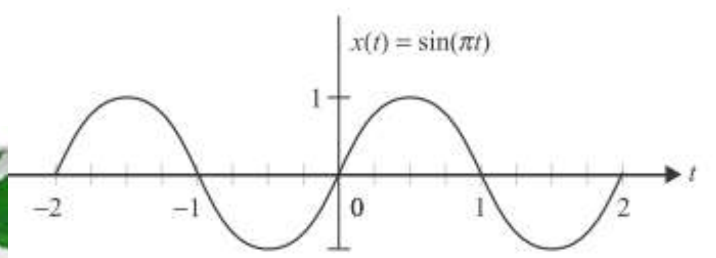
Solution

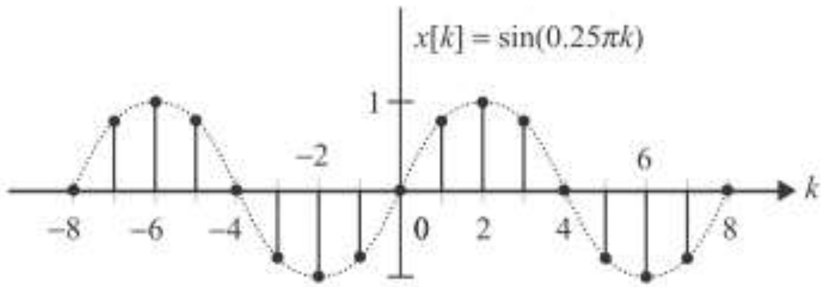
By substituting $t = kT$, the DT representation of the CT signal $x(t)$ is given by

$$x[kT] = \sin(\pi k \times T) = \sin(0.25\pi k).$$

Substitute For $k = 0, \pm1, \pm2, \ldots,$

Plotted as a function of k, the waveform for the DT signal $x[k]$ is shown





CLASSIFICATION OF SIGNALS

3. Analog and digital signals

- A second classification of signals is based on their **amplitudes**
- A signal whose amplitude can take on any value in a continuous range is an analog signal.
- This means that an analog signal amplitude can take on an infinite number of values.
- The amplitudes of many real-world signals, such as voltage, current, temperature, and pressure are analog

- **Digital signals**, on the other hand, can only have a finite number of amplitude values.
- Signals associated with typical digital devices take on only two values (binary signals).
- A digital signal whose amplitudes can take on L values is an L-ary signal of which binary (L = 2) is a special case.

CLASSIFICATION OF SIGNALS

3. Analog and digital signals

- ✓ The terms “continuous-time” and “discrete-time” qualify the nature of a signal along the time (**horizontal**) axis.
- ✓ The terms “analog” and “digital,” on the other hand, qualify the nature of the signal amplitude (**vertical**) axis.

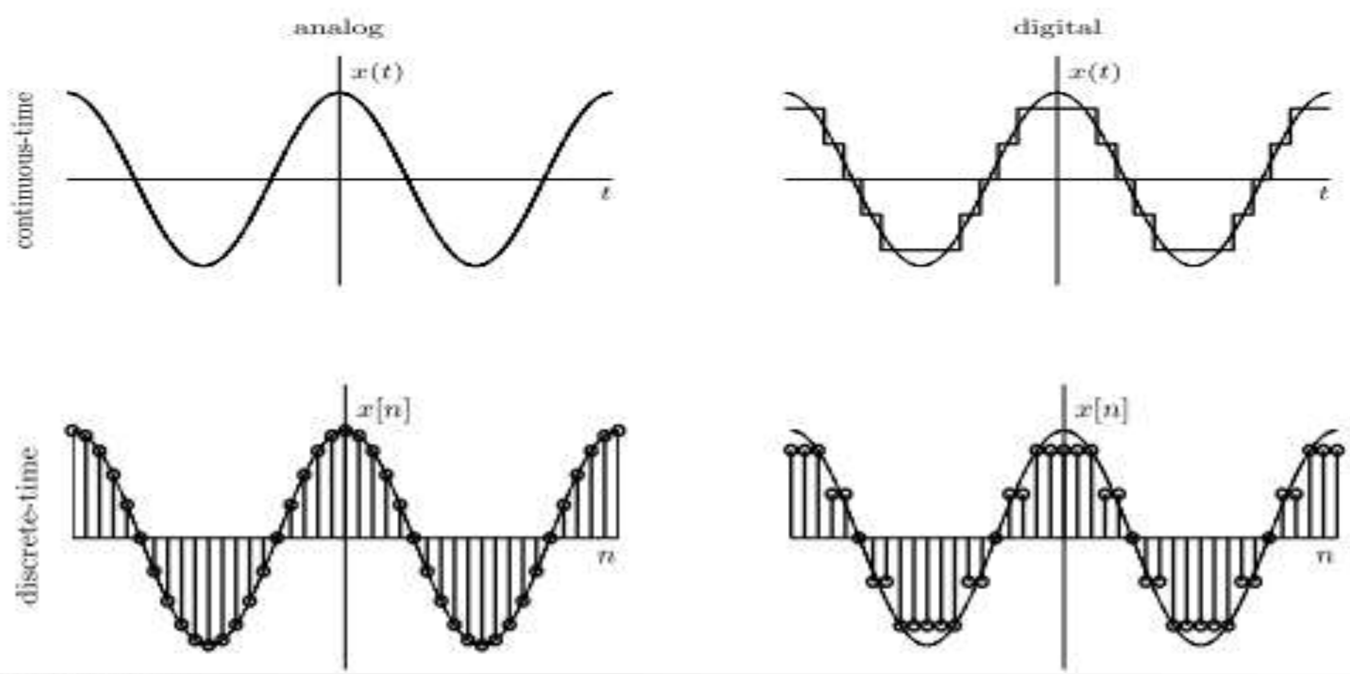
It is clear that analog is not necessarily continuous-time and that digital need not be discrete-time.

Analog, CT

Digital, CT

Analog, DT

Digital, DT



4. Periodic and aperiodic signals

A CT signal $x(t)$ is said to be *periodic* if it satisfies the following property:

$$x(t) = x(t + T_0), \qquad (1.2)$$

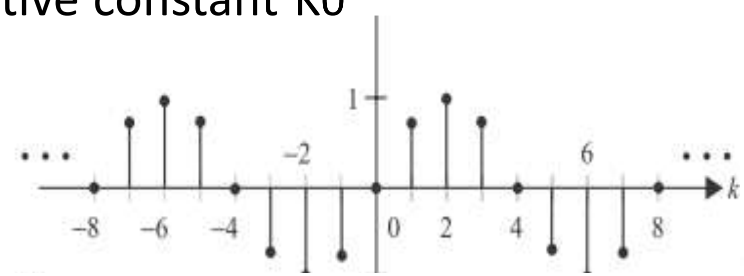
at all time t and for some positive constant T_0 .

✓ The smallest positive value is referred to as the fundamental period of x(t).

Likewise, a DT signal $x[k]$ is said to be *periodic* if it satisfies

$$x[k] = x[k + K_0] \qquad (1.3)$$

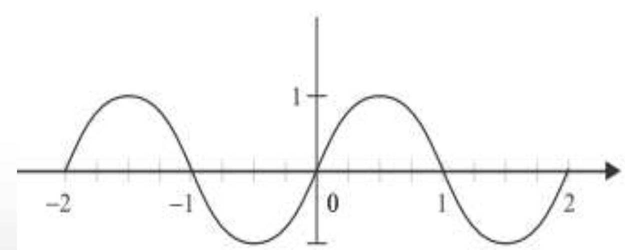
at all time k and for some positive constant K_0



- **The reciprocal of the fundamental period of a signal is called the fundamental frequency.**

$$f_0 = \frac{1}{T_0}, \text{ for CT signals,} \quad \text{or} \quad f_0 = \frac{1}{K_0}, \text{ for DT signals,}$$

$$\omega_0 = \frac{2\pi}{T_0}, \text{ for CT signals,} \quad \text{or} \quad \Omega_0 = \frac{2\pi}{K_0}, \text{ for DT signals.}$$



CLASSIFICATION OF SIGNALS

4. Periodic and aperiodic signals

✓ A familiar example of a periodic signal is a sinusoidal function

$$x(t) = A\sin(\omega_0 t + \theta).$$

Substituting t by $t + T_0$ in the sinusoidal function, yields

$$x(t + T_0) = A\sin(\omega_0 t + \omega_0 T_0 + \theta).$$

And Since :

$$x(t) = A\sin(\omega_0 t + \theta) = A\sin(\omega_0 t + 2m\pi + \theta) \qquad \text{for } m = 0, \pm 1, \pm 2, \ldots,$$

the above two expressions are equal iff $\omega_0 T_0 = 2m\pi$.

➢ The sinusoidal signal x(t) has a fundamental period :

$$T_0 = 2\pi/\omega_0$$

For m =1

➢ **A signal that is not periodic is called an aperiodic or non-periodic signal.**

4. Periodic and aperiodic signals

✓ Although all CT sinusoidal are periodic, their DT counterparts x[k] may not always be periodic.

$$x[k] = A\sin(\Omega_0 k + \theta)$$

Derive a condition for the DT sinusoidal x[k] to be periodic.

Assuming $x[k] = A\sin(\Omega_0 k + \theta)$ is periodic with period K_0 yields

$$x[k + K_0] = \sin(\Omega_0(k + K_0) + \theta) = \sin(\boxed{\Omega_0 k} + \Omega_0 K_0) + \boxed{\theta}).$$

Since x[k] can be expressed as: $x[k] = \sin(\boxed{\Omega_0 k} + 2m\pi + \boxed{\theta}),$

So the value of the fundamental period is given by:

$$K_0 = \frac{2\pi}{\Omega_0} m.$$

Since it is a DT sequences, the value of the fundamental period K0 must be an integer.

Proposition 1.1 *An arbitrary DT sinusoidal sequence $x[k] = A\sin(\Omega_0 k + \theta)$ is periodic iff $\Omega_0/2\pi$ is a rational number.*

$$\boxed{\frac{\Omega_0}{2\pi} = \frac{m}{K_0}}$$

4. Periodic and aperiodic signals

Example 1.4

Determine if the sinusoidal DT sequences (i)–(iv) are periodic:

(i) $f[k] = \sin(\pi k/12 + \pi/4)$;

Solution

The value of Ω_0 in $f[k]$ is $\pi/12$.

Since $\Omega_0/2\pi = 1/24$ is a rational number, the DT sequence $f[k]$ is periodic.

$$K_0 = \frac{2\pi}{\Omega_0} m = 24m.$$

Setting $m = 1$ yields the fundamental period $K_0 = 24$.

(iii) $h[k] = \cos(0.5k + \phi)$;

The value of Ω_0 in $h[k]$ is 0.5.

Since $\Omega_0/2\pi = 1/4\pi$ is not a rational number, the DT sequence $h[k]$ is not periodic.